Introduction
It wasn’t too long ago that the concept of factors in investing was the exclusive province of professors of finance and a few active “quant” managers. Mainstream portfolio construction was focused primarily on asset allocation. Within equities, that meant achieving the right balance in allocation to various segments such as large cap and small cap, country and sector, and perhaps value and growth style. Today, factor allocation has entered the mainstream as a complementary approach to portfolio construction, alongside traditional asset allocation. An important driver of this development has been the creation of a new array of indexes that sharply focus on one factor at a time. This has opened up new possibilities for asset owners and advisors, including investing in index-replicating financial products, both to seek a desired factor exposure at low cost and to benchmark active managers to assess the value of their fees.

One thing that followers of single-factor indexes quickly realize is that the payoff for exposure to any one factor is highly variable. Factors typically follow different return patterns: value usually exhibits pro-cyclical performance, while quality is often countercyclical, for example. Market participants who do not employ a factor-timing or factor-rotation strategy are increasingly looking at fixed combinations of factors to gain potential improvements in risk-adjusted outcomes as compared to single-factor outcomes.

This paper compares three methods for constructing multifactor indexes. We will rank them as “good,” “better” and “best” based on their factor exposure strength. Spoiler alert: we will conclude that the FTSE Russell sequential tilting or “tilt-tilt” approach is the best alternative for strength of factor capture. For reference, this paper’s appendix contains a summary of the construction of the single-factor FTSE Russell Global Factor series.

The evolution of multifactor indexes: diversification without factor dilution
A lot of the discussion concerning factor combinations seems to focus either on reductions in tracking error with respect to a broad cap-weighted benchmark or on reductions in turnover, or both. As well, since not all factors underperform at the same time, a lot of timing risk can be eliminated by combining factors. But there is a legitimate concern about diluting individual factor exposures in the construction of a multifactor index. In this section, we will review three approaches to combining factors and illustrate them with a simple three-stock example.

Good: A composite index. The first step – the simplest multifactor index – is to take a weighted average of two single-factor indexes, say 50% value and 50% quality. Let’s call that a “composite index.” The advantage of this approach is its top-down simplicity. In principle, this is no different than replicating single-factor indexes in the chosen weights. An advantage in having both factors together in one index is that the index provider maintains the fixed weights, relieving the market participant of having to adjust index-replicating products.

Better: Composite factors. The next step in the evolution has been to combine a weighted average of the individual factors in a bottom-up approach. This takes better advantage of the interaction between factors. It also offers potential trading economies. If a stock is eliminated from inclusion in one factor but added to another factor, then no trade needs to take place to maintain the index replication. Two trades would need to take place in the composite index approach.

1 Bender, Jennifer, “The Whole is Not the Sum of the Parts,” State Street Global Advisors (2015).
**Best: The tilt-tilt approach.** The most recent step in multifactor index evolution is to construct the index as a tilt of one factor on another, rather than as an averaging of the factors. This multiplicative approach, also called sequential tilting, has the best chance of achieving multifactor objectives, as we will show.

**A three-stock example.** We will illustrate the three approaches to making a quality and value multifactor index using just three stocks. First we create a hypothetical capitalization-weighted index, plus single-factor quality and value indexes for later reference. Then we illustrate the three ways of combining these two factors into a hypothetical multifactor index. We base the capitalization weights on the actual capitalization values as at June 30, 2015. Likewise, the value and quality scores are the actual scores for these stocks as at June 30, 2015.

The first column of numbers in Table 1 shows the cap-weighted index based on the actual capitalization of these stocks as at June 30, 2015, adjusted for float. The value scores calculated as at June 30, 2015 are a metric from 0 to 1, with 1 indicating a strongly value stock and 0 a strongly growth stock. We can see that Ford is strongly value, while Facebook is clearly a growth stock. Costco is exactly in the middle, neither value nor growth. We next multiply the cap weight of each stock by its value score to get the unadjusted weights of a single-factor value index. We then divide the unadjusted weights by the sum 36.8% to produce final weights that sum to 100%.

<table>
<thead>
<tr>
<th>Cap weight</th>
<th>X Value score</th>
<th>Final normalized value weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ford Motor</td>
<td>22.8% X 1.00</td>
<td>22.8%</td>
</tr>
<tr>
<td>Costco</td>
<td>20.0% X 0.50</td>
<td>10.0%</td>
</tr>
<tr>
<td>Facebook</td>
<td>57.2% X 0.07</td>
<td>4.0%</td>
</tr>
<tr>
<td><strong>SUM</strong></td>
<td><strong>100.0%</strong></td>
<td><strong>36.8%</strong></td>
</tr>
</tbody>
</table>

Source: FTSE Russell. Data as at June 30, 2015. This table is purely for illustrative purposes and does not reflect the constitution or performance of a FTSE Russell index.

In Table 2 we construct a hypothetical single-factor quality index in exactly the same way. Quality is based on measures of profitability, efficiency, earnings stability and leverage. The quality scores reveal that Costco is the highest-quality stock in this three-stock universe, with Facebook not far behind. This results in final quality weights that are quite different from the value weights.

<table>
<thead>
<tr>
<th>Cap weight</th>
<th>X Quality score</th>
<th>Final normalized quality weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ford Motor</td>
<td>22.8% X 0.26</td>
<td>5.9%</td>
</tr>
<tr>
<td>Costco</td>
<td>20.0% X 0.68</td>
<td>13.6%</td>
</tr>
<tr>
<td>Facebook</td>
<td>57.2% X 0.42</td>
<td>24.0%</td>
</tr>
<tr>
<td><strong>SUM</strong></td>
<td><strong>100.0%</strong></td>
<td><strong>43.6%</strong></td>
</tr>
</tbody>
</table>

Source: FTSE Russell. Data as at June 30, 2015. This table is purely for illustrative purposes and does not reflect the constitution or performance of a FTSE Russell index.

2 See Appendix for details on how this metric is created.
In Table 3 we show the construction of a hypothetical composite index. We assume equal weighting, but in principle, one could choose unequal weights. The composite index weights are given in the last column.

**Table 3. Creation of a composite index**

<table>
<thead>
<tr>
<th>Cap weights</th>
<th>X (Value weight + quality weight)/2</th>
<th>= Value + quality composite index weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ford Motor</td>
<td>(61.9% + 13.6%)/2</td>
<td>= 37.7%</td>
</tr>
<tr>
<td>Costco</td>
<td>(27.2% + 31.2%)/2</td>
<td>= 29.2%</td>
</tr>
<tr>
<td>Facebook</td>
<td>(10.9% + 55.2%)/2</td>
<td>= 33.0%</td>
</tr>
<tr>
<td>SUM</td>
<td>100.0%</td>
<td>100.0%</td>
</tr>
</tbody>
</table>

Source: FTSE Russell. Data as at June 30, 2015. This table is purely for illustrative purposes and does not reflect the constitution or performance of a FTSE Russell index.

Table 4 shows the construction of a hypothetical composite factor index. The value and quality scores are equal weighted and then multiplied against the cap weight to produce unadjusted weights. They are then divided by the sum 58.2% to produce weights that sum to 100%.

**Table 4. Creation of a composite factor index**

<table>
<thead>
<tr>
<th>Cap weights</th>
<th>X (Value score + quality score)/2</th>
<th>= Unadjusted weights</th>
<th>Final normalized composite factor weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ford Motor</td>
<td>22.8% X (1.00 + 0.26)/2</td>
<td>= 20.8%</td>
<td>35.7%</td>
</tr>
<tr>
<td>Costco</td>
<td>20.0% X (0.50 + 0.68)/2</td>
<td>= 17.1%</td>
<td>29.4%</td>
</tr>
<tr>
<td>Facebook</td>
<td>57.2% X (0.07 + 0.42)/2</td>
<td>= 20.3%</td>
<td>34.9%</td>
</tr>
<tr>
<td>SUM</td>
<td>100.0%</td>
<td>= 58.2%</td>
<td>100.0%</td>
</tr>
</tbody>
</table>

Source: FTSE Russell. Data as at June 30, 2015. This table is purely for illustrative purposes and does not reflect the constitution or performance of a FTSE Russell index.

Table 5 shows the construction of a hypothetical tilt-tilt value and quality multifactor index. The scores are multiplied rather than averaged. The unadjusted weights are divided by the sum 14.4% to produce the final tilt-tilt weights that sum to 100%.

**Table 5. Creation of a tilt-tilt multifactor index**

<table>
<thead>
<tr>
<th>Cap weights</th>
<th>X Value score</th>
<th>X Quality score</th>
<th>= Unadjusted weights</th>
<th>Final normalized tilt-tilt multifactor weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ford Motor</td>
<td>22.8% X 1.00</td>
<td>X 0.26</td>
<td>= 5.6%</td>
<td>41.1%</td>
</tr>
<tr>
<td>Costco</td>
<td>20.0% X 0.50</td>
<td>X 0.68</td>
<td>= 6.8%</td>
<td>47.2%</td>
</tr>
<tr>
<td>Facebook</td>
<td>57.2% X 0.07</td>
<td>X 0.42</td>
<td>= 1.7%</td>
<td>11.7%</td>
</tr>
<tr>
<td>SUM</td>
<td>100.0%</td>
<td></td>
<td>14.4%</td>
<td>100.0%</td>
</tr>
</tbody>
</table>

Source: FTSE Russell. Data as at June 30, 2015. This table is purely for illustrative purposes and does not reflect the constitution or performance of a FTSE Russell index.
We have now gone through the simple mechanics of constructing these hypothetical indexes, and we now have three sets of value and quality multifactor weights. So what difference does it make? Table 6 summarizes the active weights, with the capitalization weights backed out. In this form, the contrast between the tilt-tilt methodology and the two composite approaches is brought out clearly. The active weights of the two composite methods for Costco are much smaller than the tilt-tilt active weight. As well, the underweight to Facebook in the tilt-tilt index is much larger than with the composite approaches.

Table 6. Active weights of value-quality multifactor indexes

<table>
<thead>
<tr>
<th></th>
<th>Composite index</th>
<th>Composite factors</th>
<th>Tilt-tilt index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ford Motor</td>
<td>15.0%</td>
<td>12.9%</td>
<td>18.3%</td>
</tr>
<tr>
<td>Costco</td>
<td>9.2%</td>
<td>9.4%</td>
<td>27.2%</td>
</tr>
<tr>
<td>Facebook</td>
<td>-24.2%</td>
<td>-22.3%</td>
<td>-45.6%</td>
</tr>
<tr>
<td>SUM</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
</tr>
</tbody>
</table>

Source: FTSE Russell. Data as at June 30, 2015. This table is purely for illustrative purposes and does not reflect the constitution or performance of a FTSE Russell index.

The difference the weightings make is evaluated by the “active exposures” of the factors within each index, i.e., exposures to the factors over what naturally comes with a cap-weighted index. The value and quality scores, which are a 0 to 1 cumulative normal metric, are converted back to their underlying factor Z-scores and weighted by the active weights:

\[
\text{Active factor exposure} = \sum_{i=1}^{3} (w_i - cw_i) \cdot z_i
\]

Chart 1 displays the active exposures of our simple hypothetical three-stock indexes. Note that the active exposures of the tilt-tilt index are greater than the active exposures of either of the composite indexes for both value and quality. This is just a three-stock example, of course, and as such might not be very meaningful – except that these results qualitatively generalize to whole stock universes, as we shall see.

Chart 1. Active Value and Quality Exposures of Single Factor and Multifactor Indexes (three-stock example)

Source: FTSE Russell. Data as at June 30, 2015. This chart is purely for illustrative purposes and does not reflect the constitution or performance of a FTSE Russell index.

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Source: FTSE Russell. Data as at June 30, 2015. This table is purely for illustrative purposes and does not reflect the constitution or performance of a FTSE Russell index.
Tilt-tilt versus composite indexes applied to a whole stock universe

We employ the FTSE Developed Index universe of stocks to generalize the previous example. The universe includes the top 90% in capitalization weight of all listed stocks in all developed countries. We wish to create a combination of (high) quality, (low) volatility, and value factors. In the combination, the quality and volatility factors are positively correlated and are together often referred to as a “defensive” combination, skewing weights to the higher-quality, less-volatile corner of a stock universe. Value tends to be negatively correlated with the other two factors. We construct two hypothetical multifactor indexes: a composite index and a multiple tilt-tilt index.3

The weights for each stock \( i \) in the quality-volatility-value composite index would be:

\[
W_{\text{Composite},i} = \frac{W_{\text{Quality},i} + W_{\text{Volatility},i} + W_{\text{Value},i}}{3},
\]

where the right-hand-side weights are from single-factor indexes. The multiple tilt-tilt (unadjusted) index weights are:

\[
\text{Unadjusted}W_{\text{Tilt},i} = \text{QualityScore}_i \times \text{VolatilityScore}_i \times \text{ValueScore}_i \times \text{CapWeight}_i,
\]

Recall that the scores range from zero to one from a cumulative normal distribution mapping from the factor Z-scores. The adjusted weight is the unadjusted weight normalized to sum to 100%:

\[
W_{\text{Tilt},i} = \frac{\text{Unadjusted}W_{\text{Tilt},i}}{\sum \text{Unadjusted}W_{\text{Tilt},i}}
\]

Charts 2, 3 and 4 display the hypothetical average active exposures over the period September 2001 through July 2015. Active exposures are measured in the same way as in the three-stock example, the active weighted average of the factor Z-scores. All three charts share the same pattern, namely that while both multifactor indexes have reduced exposures compared to single-factor indexes, the multiple tilt approach gives up less than the composite index method.

Chart 2. Active Value Factor Exposures

![Chart 2. Active Value Factor Exposures](image)

Source: FTSE Russell. Data as at July 31, 2015. This chart is purely for illustrative purposes and does not reflect the constitution or performance of a FTSE Russell index.

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3 We do not look at the composite factor approach, in the interest of conserving space. As well, it is our observation that the composite index approach is the most frequently cited method of combining factors.
We can see that the multiple tilt exposures are visually larger than the composite exposures, but are the differences statistically significant? To find out, we conducted pairwise t-tests on the null hypothesis that the monthly active factor exposures in the multiple tilt and composite indexes were on average no different. Table 7 shows that the null hypothesis is rejected for all three factors, i.e., the differences in factor exposures are indeed statistically significant.

**Table 7. Pairwise t-tests of differences in factor exposures**

<table>
<thead>
<tr>
<th></th>
<th>Mean difference in exposures: multiple tilt vs. composite index</th>
<th>T-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>0.31</td>
<td>52.67</td>
</tr>
<tr>
<td>Quality</td>
<td>0.26</td>
<td>45.90</td>
</tr>
<tr>
<td>Volatility</td>
<td>0.31</td>
<td>57.11</td>
</tr>
</tbody>
</table>

Source: FTSE Russell. Data as at July 31, 2015. This chart is purely for illustrative purposes and does not reflect the constitution or performance of a FTSE Russell index.
Chart 5 shows how the differences in exposures between the multiple tilt and composite indexes would have manifested in performance. The dilution of the factor exposures in the composite construction would have resulted in an index that is barely distinguishable from the cap-weighted FTSE Developed Index.

![Chart 5. Quality, Volatility and Value Tilt-Tilt versus Composite Performance](image)

Source: FTSE Russell. Data as at July 31, 2015. Past performance is no guarantee of future results. Returns shown reflect FTSE Developed Index and hypothetical historical performance. Data for the hypothetical multiple tilt and composite indexes is purely for illustrative purposes and does not reflect the constitution or performance of a FTSE Russell index. Please see the final page for important legal disclosures.

Moreover, the gap in exposures between tilt-tilt and composites increases directly with the number of factors, as Charts 6, 7 and 8 show. Using the FTSE Developed Index universe, the charts show the percentage of single-factor exposures that are captured by multifactor combinations. As more factors are added, the proportion of single-factor exposures captured steadily decreases for the composite index approach. That’s because a linear combination of factors tends toward dilution of factor strength. The nonlinear combination of factors in the tilt-tilt approach ensures that the proportion of single-factor strength captured remains steady, or even increases in some cases.
Chart 6. Value + Momentum

Source: FTSE Russell. Data as at July 31, 2015. This chart is purely for illustrative purposes and does not reflect the constitution or performance of a FTSE Russell index.

Chart 7. Value + Momentum + Quality

Source: FTSE Russell. Data as at July 31, 2015. This chart is purely for illustrative purposes and does not reflect the constitution or performance of a FTSE Russell index.
Visualizing the tilt-tilt and composite approaches

Chart 9 is an aid to visualizing the difference between tilt-tilt and composite indexing through the lens of double sorting, which is used extensively in academic research. Consider the value and quality dimensions in Chart 9. The double sort method separates a stock universe into sub-portfolios based on quantiles of the factor Z-scores. There are nine sub-portfolios based on tri-tiles of each factor in this chart, but in practice there are typically more—say, 25 sub-portfolios based on quintiles. The yellow box in the lower right corner is an intersection sub-portfolio that simultaneously exhibits strong factor capture in both factors, which is analogous to the tilt-tilt method. This is in contrast to a blended approach that would combine single-factor sub-portfolios as a blended sub-portfolio (visible as the five boxes with red borders), which is analogous to a composite index approach. Recent research has concluded that simultaneous sub-portfolios are “superior” to blended sub-portfolios in capturing risk premia.4

Using standard double sorting to create an index would have limitations. The sub-portfolios are “lumpy,” in the sense that a stock in the upper left corner of the yellow box, with the weakest factor capture within the box, would have the same weight as a stock in the lower right corner, with the strongest factor capture. One can mitigate this by taking ever-finer quantiles, but at some point one would have sub-portfolios with only one stock in each. Then it becomes very close to the tilt-tilt approach as illustrated in Chart 10, which is a continuous and simultaneous capture of the two factors. Figuratively speaking, the tilt-tilt index is then formed by drawing an arc like one of those shown in Chart 10, striving for strong factor capture while balancing concerns about liquidity, capacity, diversification and turnover.

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Conclusion: A holistic approach to factor investing

A multifactor index should embrace a holistic approach by targeting multiple outcomes simultaneously, rather than approaching each individual factor component separately. And it should do so in such a way that the dilution of the competing factor objectives is kept to a minimum. Simply averaging factor indexes or averaging factors can only satisfy preferences for weaker exposure to all target factors.

We believe the FTSE Russell sequential tilting (“tilt-tilt”) approach provides an effective and general means of pursuing multiple factor objectives via strong factor capture, and is therefore a powerful mechanism for implementing factor objectives.

Appendix: Single-factor index construction

Factors are variables that drive equity returns. These variables are common to all stocks or a group of stocks, and cannot be easily diversified away. The relevant individual factors, their numbers and their definitions vary somewhat from one index provider to the next, but they are typically based on a broad academic and practitioner consensus. In its Global Factor Index series, FTSE Russell uses the following definitions for eight single-factor measures:

- **Value**: Combination of trailing cash-flow yield, earnings yield and country-relative sales-to-price ratio
- **Size**: Natural logarithm of full market capitalization
- **Momentum**: Total return in local currency terms over the previous year
- **Residual momentum**: Sharpe ratio of residuals after backing out country and global industry returns
- **Volatility**: Standard deviation of 5 years of weekly (Wednesday to Wednesday) local total returns
- **Quality**: Combination of profitability (return on assets), efficiency (change in asset turnover), earnings quality (accruals) and leverage
- **Liquidity**: Log of the Amihud ratio (the median ratio of absolute daily return to daily traded value) over the previous year
- **Yield**: Natural logarithm of each company’s 12-month trailing dividend yield
There remains the question of how to map the above definitions into factor exposures in an index form. The factor index should provide a strong but controlled exposure to the factor by use of a common, transparent and rules-based methodology. In order to serve as both a benchmark for particular factor strategies and the basis for index-replicating financial products, the methodology needs to pay attention to liquidity, capacity, diversification and turnover. There is often a trade-off between these objectives.

The FTSE Russell–FTSE Global Factor Index series follows two design steps intended to strike a balance between these objectives:

- Factor scores are standardized to fall within a range of +/- 3 standard deviations from the mean factor score of zero
- Factor scores are converted into index weights that range from 0 to 1 using a cumulative normal distribution function

Together, these two steps have the effect of limiting the impact of the smallest and largest factor scores while avoiding an extreme concentration of the index in a few stocks with high factor scores. The final step multiplies the factor score with the capitalization weight to produce the factor index weights. The process is summarized in the accompanying illustration.

### Constructing single-factor indexes

1. **Calculate factor Z-scores**
   - Calculate standardized factor score (Z-score) as $\frac{\text{[factor score-mean]}}{\text{standard deviation}}$
   - Set maximum Z-scores as +/- 3

2. **Map Z-scores to scores**
   - Use cumulative normal mapping function to assign scores $S_i$ (0 <= $S_i$ <= 1) to individual Z-scores

3. **Translate scores to index weights**
   - Multiply weights $W_i$ in starting index by scores $S_i$ to produce factor index weights

Finally, factor capture is strengthened by universe truncation or “narrowing,” i.e., by removing the stocks with the smallest contribution to the index’s factor exposure. This is done sequentially to maximize exposures while satisfying capacity, turnover and sector diversification constraints.
About FTSE Russell

FTSE Russell is a leading global provider of benchmarking, analytics and data solutions for investors, giving them a precise view of the market relevant to their investment process. A comprehensive range of reliable and accurate indexes provides investors worldwide with the tools they require to measure and benchmark markets across asset classes, styles or strategies.

FTSE Russell index expertise and products are used extensively by institutional and retail investors globally. For over 30 years, leading asset owners, asset managers, ETF providers and investment banks have chosen FTSE Russell indexes to benchmark their investment performance and create ETFs, structured products and index-based derivatives.

FTSE Russell is focused on applying the highest industry standards in index design and governance, employing transparent rules-based methodology informed by independent committees of leading market participants. FTSE Russell fully embraces the IOSCO Principles and its Statement of Compliance has received independent assurance. Index innovation is driven by client needs and customer partnerships, allowing FTSE Russell to continually enhance the breadth, depth and reach of its offering.

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