CoCo (contingent conversion) bonds have seen an upsurge in the headlines lately. In a nutshell, these instruments allow banks to boost regulatory capital during periods of financial stress, but not at the expense of taxpayers; hence, these instruments mitigate the too-big-to-fail doctrine. Investors of CoCos take the brunt of losses if a bank’s capital ratio dips below a predefined level. When Common Equity Tier 1 (CET1) capital falls below a preset threshold level, the bonds either convert into cheap equity shares or suffer a partial or full write-down in principal. In either case, this loss-absorption mechanism immediately strengthens the capital structure of the issuing bank. For taking on this “regulatory” risk, investors are compensated with higher yields, which are attractive under the current low interest rate environment.

Conflicting signals

From the onset, CoCos are appealing from both a regulator’s and bank’s perspective because they inject capital before insolvency without a government bailout. However, Cocos as an asset class have recently come under scrutiny, not only from investors, as might be expected, but from an issuing bank. In particular, Deutsche Bank (DB) in March of this year stated that CoCos were bad products and that DB might stop issuing them altogether. But these statements did not discourage investor demand for CoCos. In April, just one month later, UBS issued 1.5B USD worth of CoCos. In addition, the Spanish bank Banco Bilbao Vizcaya Argentaria (BBVA) and the Dutch bank ING both issued 1B EUR worth of CoCos. In all cases demand outstripped supply. Why this wide disconnect among issuer’s perceptions of contingent capital issuance? The negative statements about CoCos from DB might sound surprising, but these came after a sell-off in bank shares and bonds in February, with a partial recovery in March;
see Figure 1. Further, DB faced litigation for manipulating LIBOR and was restructuring to focus on its core business. In fact, investors of DB CoCos worried less about the trigger event than about missed coupon payments because of insufficient bank reserves. DB even had to reassure its investors that it had sufficient funds to pay coupons. The potential of missed coupons is another key risk that investors must consider. As of the writing of this note, no CoCo has experienced a missed coupon or trigger event.

The bank sell-off in February was due to poor earnings statements in the banking sector. For example, DB posted its largest loss since the financial crisis in 2008, and Credit Suisse had its largest full-year loss since the crisis. Prior to the sell-off, the European Banking Authority (EBA) in December released a note suggesting that coupons might be canceled earlier than expected. The EBA recommended that banks should disclose capital requirements that could limit their ability to pay CoCo coupons (along with bonuses and dividends). It might be a stretch to say that this contributed to the sell-off, but nonetheless it is interesting that a regulatory agency had concerns about missed coupon payments.

Figure 1: Cumulative equity returns during bank sell-off
MECHANICS OF COCOS

The goal of CoCos is to readily provide bank capital during a financial crisis (prior to insolvency) at the expense of the investors. All CoCos are characterized with a loss-absorption mechanism and a trigger that activates that mechanism:

- **Loss-absorption mechanism**: Losses should be absorbed prior to the point of insolvency. After the trigger event, the CoCo is either converted to equity or suffers a principal write-down. Lately, write-down CoCos have been popular since they will not dilute existing shares.

- **Trigger levels**: Activation of losses is triggered when the issuing bank’s capital falls below a specified fraction of its risk-weighted assets (RWAs). More precisely, when the ratio of CET1 capital to RWA falls below a threshold, \( t^* \), CoCo investors face losses either through conversion to equity or a write-down:

\[
\frac{\text{CET1}}{\text{RWA}} < t^*,
\]

We can represent the loss, \( L \), of a CoCo as

\[
L = N - CR \cdot S^* \quad \text{(2)}
\]

\[
= N \left(1 - \frac{S^*}{X}\right), \quad \text{(3)}
\]

where \( N \) is the notional, \( CR \) is the conversion ratio, \( X = N/CR \) is the conversion price, and \( S^* \) is the (unknown) equity value when the capital ratio is breached. The last term in (3) can be interpreted as a recovery rate; here the conversion price determines the loss when a trigger event occurs. For example, if \( X = S^* \) then no loss is observed. In fact, the loss-absorption mechanism is not standardized and varies in the CoCo market. Some common specifications are: (i) \( X = S_0 \) where \( S_0 \) is the initial equity price at CoCo issuance; (ii) \( X = \theta S_0 \) where \( \theta \) is a specified fraction of the issue price; and (iii) \( X = \min(S_0, S_1) \) with a specified floor share price of \( S_1 \). For write-down CoCos, we can replace the last term in (3) with a recovery rate that reflects either full or partial principal write-downs.

The predefined accounting trigger (1) is typically accompanied with a **regulatory trigger** at the discretion of the local supervisor. The discretionary trigger is also known as a point of non-viability (PONV) trigger, which allows regulators to activate the loss-absorption if they believe a bank is close to insolvency. It allows regulators to act more quickly than accounting data would permit, but at the cost of adding uncertainty around the timing of the trigger event.
Regulatory fine-tuning

CoCos are a relatively new asset class that first appeared in 2009 when Lloyds Bank issued its equity credit notes (ECNs). Since their inception to strengthen the financial health of the banking sector, CoCos have come under scrutiny by regulators. Regulatory bodies such as the European Securities and Market Authority (ESMA) and Germany’s Federal Financial Supervisory Authority (BaFin) have advised that only sophisticated investors such as asset managers and hedge funds should be allowed to purchase CoCos. Because CoCos carry many risks (which we will discuss in the next section), this is a prudent measure to protect retail investors.

Figure 2: CoCos within the capital structure of a bank

Moreover, there have been changes to the regulatory framework on how CoCos should be structured to qualify as regulatory capital. Under Basel III, CoCos can qualify as either Additional Tier 1 (AT1) or Tier 2 (T2) capital; see Figure 2. T2 capital, which sits below senior debt and deposits but above T1 capital, is gone-concern capital—i.e., it ensures loss-absorption during liquidation. To qualify as AT1 capital, CoCos must be going-concern contingent capital—i.e., it helps absorb losses and prevent insolvency. In this case, a bank can continue to operate on an ongoing basis. To qualify as T1, the trigger level needs to be set at or above the regulatory requirement, which is currently 5.125%.[4] In addition, AT1 CoCos must

---

[4] The minimum level is 5.125% for 2016 and is expected to increase incrementally to 7% by 2019.
Callable AT1 CoCos are required to have the first call date at least five years after issuance; see [1] for details. Further, step-up or other incentives to redeem are no longer permitted. In the past, step-up coupons resulting in investors’ anticipation of redemption at the first call date.

**EMBEDDED RISKS**

CoCos are hybrid securities that exhibit both bond- and equity-like features. In the capital structure of the bank, they sit between senior debt and equity, and qualify as either T2 or AT1 capital. (See Figure 2.) AT1 CoCos are the most subordinated debt and consequently the most risky bank debt.

CoCos carry a wide spectrum of risk, including market, credit, and regulatory risk. (See Table 1.) As a result, the valuation and risk modeling of these complex securities is difficult. First, modeling the trigger event itself is challenging. The accounting trigger contains in its denominator the value of RWA, which is a function of a bank’s internal models, reported at most on a quarterly basis. Thus, different banks compute RWA differently for identical portfolios of assets. As a result, the trigger may not accurately indicate impeding insolvency. Moreover, because the accounting triggers are lagged, they will not provide an early warning during a bank crisis.

Extension and missed-coupon risk for AT1 capital is important to acknowledge. As described earlier, callable AT1 CoCos must have the first call date at least five years after issuance. After the first call, the coupon payments typically switch from fixed to float. Thus, modeling assumptions about redemption are required to compute yields and spreads for CoCos. One can compute yields by using a simple approach such as yield-to-worst or -first or from a standard interest rate model that handles arbitrary coupon structures. Missed-coupon or coupon cancellation risk is the risk of the bank not paying interest to CoCo investors. Recall that Deutsche Bank’s perceived inability to make future coupon payments after the February sell-off caused turmoil for its contingent capital investors.

Finally, there is regulatory risk. Recall that a discretionary trigger may accompany an accounting trigger. Anticipating when regulators will use a discretionary trigger or when they will permit redemption, even if it makes sense economically from the bank’s perspective, is virtually impossible. In addition, when regulatory capital definitions change, the issuer can call at the prevailing market price (regulatory call provision). At times, the price of CoCos has soared when investors anticipated an issuer call due to the reclassification of regulatory capital. An interesting case involves Swiss regulators in October 2015, when they proposed that only high-trigger CoCos be classified as AT1 capital. Consequently, investors
anticipated a regulatory call of a low-trigger AT1 Coco issued by Credit Suisse. This drove its price above par until the regulators decided to grandfather this particular CoCo as an AT1.

### Table 1: Embedded risks for CoCo bonds

<table>
<thead>
<tr>
<th>Key Risk</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest rates</td>
<td>Currently minimal in low-rate environment</td>
</tr>
<tr>
<td>Credit spread</td>
<td>Systematic and issuer specific</td>
</tr>
<tr>
<td>Coupon cancellation</td>
<td>AT1 capital is subject to missed coupon payments</td>
</tr>
<tr>
<td>Extension risk</td>
<td>Most AT1 CoCos are callable</td>
</tr>
<tr>
<td>Regulatory risk</td>
<td>- timing of discretionary trigger</td>
</tr>
<tr>
<td></td>
<td>- discretionary coupon cancellation</td>
</tr>
<tr>
<td></td>
<td>- extension risk</td>
</tr>
<tr>
<td></td>
<td>- regulatory call provision: issuer can call if regulatory capital definitions change</td>
</tr>
</tbody>
</table>

**WHAT'S A RISK MANAGER TO DO?**

At one extreme, one can claim that modeling CoCos adequately is impossible. As discussed above, this is due to an opaque definition of the accounting trigger (or RWA), the timing of the trigger event, missed coupon risk, and extension risk. And on top of this, we have regulatory risk that entails regulatory call provisions, discretionary triggers, and redemption approval. As a consequence, we may never be able to model all the risks of CoCos. However, we can begin with standard models, which have been proposed in the literature, for valuation and risk estimations. As a first step, it seems reasonable to link the pricing model of a CoCo to its equity. This is a reasonable starting point given that CoCos are hybrid securities. As a bank’s credit deteriorates, its share price will plummet and equity volatility will increase. The three models that we outline below, to varying extents, incorporate equity prices and volatility into
the pricing.

In this section we will outline the models and use one (a market-based structural model) to compare the creditworthiness of a sample of European banks that issue CoCos.

**Pricing models**

The pricing models for CoCos can be divided into three broad categories:

- Equity derivative models
- Credit derivative models
- Structural models

This wide spectrum of pricing models is due to the hybrid features of CoCos. For a description of equity and credit derivative models, see De Spiegeleer and Schoutens [3]. With the equity derivative model, a CoCo is modeled as a bullet bond and a series of knock-in derivatives. Under a credit derivatives approach, the standard concepts of risky bond pricing are utilized. Each bond’s cash flow has a probability of survival associated with it. The model is adjusted for the probability of a trigger event rather than default. Instead of modeling the dynamics of the accounting trigger explicitly, an implicit equity barrier (see Equation (3)) is introduced such that if the share price dips below this level, a trigger event is activated. See Appendix A for a description of the credit derivatives model.

Structural models take as inputs the asset value, asset volatility, and firm leverage, and output a credit spread. Default occurs when the asset value falls below the firm’s liability level. The dynamics of the firm’s assets drives both the firm’s equity and credit, and consequently provides a link between equity and credit markets.

Moreover, structural models can be classified as fundamental or market-based. (See Stamicar and Finger [9].) Both fundamental and market-based structural models can be extended to model CoCos. The fundamental approach estimates inputs using first principles and then computes model prices of financial instruments. The fundamental model provides a natural framework that incorporates the bank’s balance sheet. Incorporating CoCos into the capital structure is nontrivial, but several fundamental models have been proposed; see, for example, Metzler and Ressor [6] or Pennacchi [8]. On the other hand, market-based approaches observe market prices of financial instruments and solve for the model parameters that allow us to recover these prices. For example, given the price of a CoCo, we can infer the leverage that recovers its price. The market-based approach is attractive in that it can bypass the estimation of leverage.

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2Structural models provide default probabilities as outputs. Since we focus on the pricing of CoCos, we only consider risk-neutral versions of structural models.

3Market-based structural models are similar to the credit derivatives model that De Spiegeleer and Schoutens [3] proposed.
and can be used to calibrate models for risk estimations such as VaR. We will provide examples at the end of this note to illustrate this approach. Table 2 provides a list of inputs and outputs for structural models.

Table 2: Fundamental and market-based structural models

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asset value</td>
<td>Risk-neutral probabilities</td>
</tr>
<tr>
<td>Leverage</td>
<td>Credit spreads</td>
</tr>
<tr>
<td>Market cap.</td>
<td></td>
</tr>
<tr>
<td>Equity volatility</td>
<td></td>
</tr>
<tr>
<td>Capital structure</td>
<td></td>
</tr>
<tr>
<td>including CoCos</td>
<td></td>
</tr>
</tbody>
</table>

Which model?

Equity derivative, credit derivative, and market-based structural models might better address the regulatory uncertainty than do fundamental structural models. But the lagged financial statement data remains to be addressed. In fact, De Spiegeleer, Marquet and Schoutens [2] propose a model, which is structural in spirit, where the regulatory trigger is explicitly modeled. They introduce a distance-to-trigger parameter that is analogous to distance-to-default. At first, this seems of little use since accounting information lags and different banks compute RWA differently. But the authors cleverly introduce an uncertainty in this metric through a CET1 volatility parameter. This uncertainty in CET1 can potentially be added to a structural framework.

In the next few sections, we will examine how a particular market-based structural model can provide an early warning signal. The structural model that we consider is the E2C/CreditGrades model that was initially introduced by JPMorgan. (See Finger [5].) It is a first exit barrier model, which is similar to the credit derivatives model. This structural model provides tractable solutions for risk-neutral probabilities and spreads. In our market-based implementation of E2C/CreditGrades, we introduce an implied barrier that represents the leverage of the firm. See Appendix A for a description of this model. We are not claiming that market-based structural (or credit derivative) models are the best models, but we select them because implementation is fairly straightforward.

A standard model for a CoCo should:
• Link to the CoCo’s equity
• Provide a warning by using CoCo spreads to imply leverage
• Use other sources of data such as CDS or implied volatilities as a warning signal at the issuer level
• Stress test redemption assumptions

Implied leverage as an early warning signal

Recall that the accounting trigger is updated only when the bank publishes a new RWA, and as a result does not provide a reliable warning. Daily market data provides a more timely warning.

To illustrate, we use the market-based version of E2C/CreditGrades, described in Appendix A, to compute implied leverage. Leverage can be defined in various ways, and for our analysis, we define leverage simplistically as long-term debt to market capitalization. We use balance sheet information to calibrate the model to the initial value of leverage, which then changes with market data. Consider the case where our inputs are the equity price, equity volatility, and the five-year CDS spread of a bank. From these parameters, we compute an implied asset volatility that recovers the initial CDS spread and an initial leverage as described above. We keep the asset volatility fixed as equity and CDS prices change. Figure 3 provides implied leverages for a sample of European banks. During February implied leverage increased for most banks, with a notable increase for DB. At the other extreme we see UBS with the smallest change in implied leverage.

Although the implied leverage computed from CDS is useful, we have not utilized any CoCo data yet. We can incorporate equity prices and CoCo spreads into the computation of implied leverage as well. Our sample consists of AT1 CoCos with write-down triggers. Figure 4 displays the implied leverage from CoCo spreads. CoCo spread-implied leverage behaves much like CDS-implied leverage, but the movements in leverage for certain banks were much more pronounced over the periods of the bank sell-off in February and the Brexit referendum in late June.

We can interpret the differences between the CDS and CoCo spread versions of implied leverage that are illustrated in Figures 3 and 4 as follows: the CDS-implied leverage captures the creditworthiness of the issuer, whereas the CoCo spread-implied leverage is more specific to the bank’s CoCo issuance. For example, we observe much more pronounced movements in leverage for Banco Popular (POP), UnitCredit (UCG), and Banco Santander (SAN) over both the bank sell-off and Brexit periods. These results are not surprising since the CoCo bonds of these three banks suffered big losses during the bank sell-off; at times they were trading below 75 cents to the euro.
Figure 3: Implied leverage derived from CDS spreads and equity data

Figure 4: Implied leverage derived from Coco spreads and equity data
**CoCo basis**

Another useful measure is the basis between the spreads of CoCos and other risky securities from the same issuer. Basis measures allow us to perform relative value analysis between different assets and help detect dislocations in the market. Consider the basis over CDS spreads, i.e., the difference between CoCo and CDS spreads, in Figure 5. For Deutsche Bank (DB) and UBS, the CDS-basis is fairly flat, which indicates that the CoCo and CDS prices are moving in lockstep. The widening of the basis for Banco Popular (POP) and UniCredit (UCG) over the bank sell-off period in February indicates the market’s concern for CoCo risk.

In addition, we can look at other basis measures over different seniority levels of bonds. Figure 6 plots the basis between the spreads of CoCos and unsecured bonds. Here we see the basis increased significantly for DB. This is no surprise given the investor concern of missed coupon payments.

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**Figure 5: Basis between CoCo and CDS spreads**

4. Modeling assumptions about redemption are required to compute spreads. In this case we take the worst-to-yield redemption date to compute spreads.

5. We compute the bond spread for senior unsecured bonds by weighting individual bond spreads with the amount outstanding notional.
Figure 6: Basis between CoCo and bond spreads. Right-hand plot is identical to left-hand plot except that the range in the y-axis has been restricted.

**RISK ESTIMATION**

We take a pragmatic approach to estimating risk for CoCos because of the myriad of risks described earlier. In this section we apply the E2C/CreditGrades model to compute VaR for a small subset of AT1 CoCos listed in Table 3. We perform two backtests of the model in this section. The first is based on comparing realized P&Ls (profits and losses) against ex-ante risk forecasts, and the second is based on how well the model tracks realized spreads when given perfect foresight in the movement of underlying risk factors.

In Figure 7 we illustrate our first backtest. We plot daily P&Ls against predicted VaR, at the 95% confidence level and one-month decay, and monitor the number of excessions from the confidence bands. We expect to observe downside excessions 5% of the time. Over this period of daily returns, we are in line with the expected number of excessions (5-6). From the plot, we observe the increased level of risk during the bank sell-off period in February and the period prior to the Brexit referendum in late June. In Figure 8 we also plot VaR confidence bands for some CoCo bonds on a stand-alone basis. The pattern of elevated risk during the bank sell-off and Brexit periods are similar to the portfolio result in Figure 7.

In our second backtesting analysis, we compare CoCo with implied CoCo spreads. At the beginning
period of our analysis, we calibrate the leverage such that we recover the initial CoCo spread. We assume perfect foresight and allow the equity price to evolve exactly as it did over this period. The equity volatility, which is an input for the implied spreads, is estimated using a rolling six-month look-back. The results are provided in Figure 9 which shows that the implied spread tracks the realized spread well. In time, a notable basis appears between the spreads for some cases such as UniCredit, but under an ex-ante risk analysis such as VaR, we would calibrate at a more frequent frequency such as daily or weekly. Nonetheless, the plots suggest that this (relatively) simple model will capture CoCo risk at least to first order.

Although the analysis is not comprehensive, it illustrates that when combined with early warning signals, the appropriate model can be a useful risk management tool. The results of our second backtest look promising and will be extended to a larger universe.

<table>
<thead>
<tr>
<th>Issuer</th>
<th>ISIN</th>
<th>Coupon (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deutsche Bank</td>
<td>DE000DB7XHP3</td>
<td>6.00</td>
</tr>
<tr>
<td>UBS</td>
<td>CH0271428309</td>
<td>5.75</td>
</tr>
<tr>
<td>Banco Popular</td>
<td>XS0979444402</td>
<td>11.5</td>
</tr>
<tr>
<td>Banco Santander</td>
<td>XS1043535092</td>
<td>6.25</td>
</tr>
<tr>
<td>UniCredit</td>
<td>XS1107890847</td>
<td>6.75</td>
</tr>
<tr>
<td>Credit Suisse</td>
<td>CH0221803791</td>
<td>6.00</td>
</tr>
<tr>
<td>HSBC</td>
<td>XS11111123987</td>
<td>5.25</td>
</tr>
<tr>
<td>Barclays Bank</td>
<td>XS1002801758</td>
<td>8.00</td>
</tr>
</tbody>
</table>
Figure 7: Backtest for a portfolio of CoCos

Figure 8: Backtest standalone results for the sample of CoCos in Table 3
Figure 9: Implied vs. realized spreads for a sample of AT1 CoCos
CONCLUDING REMARKS

AT1 CoCos can be an important component in capital requirements. During a bank crisis, they can boost regulatory capital through their loss-absorption mechanism. Instead of a taxpayer bailout, investors absorb losses when the bond converts to equity or the principal is written off.

The valuation and modeling of these complex hybrid securities is challenging. The key risks and modeling hurdles include the timing of the trigger event, extension risk, coupon cancellation, and regulatory uncertainty about discretionary coupon cancellation and trigger events. We did not address all these risks directly, but instead focused on a market-based approach where we applied a relatively simple structural model. First, we explained how warning signals based on market data and balance sheet information can help to identify and mitigate issuer risk during stressed periods. We then examined the performance of implied leverage, which is based on market data composed of CDS spreads, equity prices, and CoCo spreads, during the bank sell-off in February and the period around the Brexit referendum. In addition, we examined some basis measures, which can be used to detect relative value opportunities and dislocations in the market. Finally, we analyzed the risk for a small sample of CoCos and compared the realized P&Ls with VaR.

One key risk that we did not address in this note is coupon cancellation risk. Although we did not

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6See De Spiegeleer, Schoutens, and Van Hulle for a treatment of coupon cancellation risk that involves a Monte Carlo framework.
include this in our model, including this analysis in a risk management framework is beneficial. The ability to create stress tests on missed coupons and redemption assumptions at different call dates complements a market-based risk analysis.

THE FUTURE OF COCOS

There have been positive steps to remove ambiguity around CoCo investing. For instance, the regulatory classification of CoCos has evolved, and currently under Basel III, there is increased transparency concerning how CoCos qualify as either T2 or AT1 capital.

However, there remains a lack of standardization for CoCos in terms of trigger levels, loss-absorption mechanisms, and necessary capital buffer levels. Greater standardization from regulators would benefit investors and potentially enhance modeling techniques. For instance, the mechanics of coupon cancellation are unclear. Providing transparency and regulation for the maximum distributable amount (MDA) that governs the proportion of earnings that can be distributed as dividends, AT1 coupon payments and bonuses would be beneficial. Regulatory-specified details of a MDA waterfall could be incorporated into a risk framework of stress testing and pricing models. Regulators could also reduce the variability on RWA calculation from bank’s internal models. This could be achieved to some extent by allowing banks to estimate their own probability of defaults, but with loss given default parameters provided by regulators.

With potentially more regulatory fine-tuning, it will be interesting to see how CoCos as an asset class will evolve. As CoCos and CoCo regulation coevolve, our approach to pricing and risk modeling will also continue to adapt in interesting ways.
APPENDIX A

Spiegeleer and Schoutens [4] present two practitioners’ approaches to pricing CoCos, both of which are based on observable market inputs. The first method is based on a credit derivatives framework while the second approach is based on an equity derivatives framework.

In this note we will focus on the credit derivatives method outlined in [3] and present an extension based on a random barrier. Since the publication of capital ratios has an inherent lag, the inclusion of an uncertain barrier from a modeling perspective seems suitable.

In addition, we will present an approach based on a structural model. It is important to point out that this version of the structural model is based on risk-neutral probabilities. In fact, the structural model that we present will be similar to the credit derivatives approach: both will be calibrated from market observables; in the case of credit derivatives an implied equity threshold is backed out while in the case of the structural model, an implied leverage is backed out.

CREDIT DERIVATIVES APPROACH

In a reduced form approach, we model default as an exogenous process. Under a single period, we can price a zero bond with unit notional as

\[ e^{-(r+s)t} = e^{-rt}(1 - p_{surv})R + p_{surv} \]  

(4)

where \( r \) is the risk-free rate, \( s \) is the bond spread, \( t \) is the time to maturity, \( p_{surv} \) is the survival probability, and \( R \) is the recovery rate. From this equation we can obtain

\[ s = (1 - R)h \]  

(5)

where \( p_{surv} = e^{-ht} \). For a CoCo, we use the same credit derivative framework, but replace the default intensity \( h \) with a trigger intensity \( h_{CoCo} \):

\[ s_{CoCo} = (1 - R_{CoCo})h_{CoCo} \]  

(6)

Loss and conversion

The conversion price is defined as

\[ X = \frac{N}{n} \]  

(7)
where $N$ is the notional and $n$ is the conversion ratio of the CoCo. For a traditional convertible bond (CB), it is in-the-money when the underlying stock price $S$ is greater than the conversion price, i.e., $S > X$, and out-of-the-money when $S < X$. Unlike a traditional convertible bond where the investor exercises conversion, a CoCo will either experience a write-down or forced conversion when the Core Tier 1 ratio drops below a threshold. This will occur at an unknown stock price $S^*$. The loss incurred by the investor is

$$L_{CoCo} = N - nS^* = N \left(1 - \frac{S^*}{X}\right)$$

Thus the recovery rate in (6) is

$$R_{CoCo} = \frac{S^*}{X}$$

**Barrier problem**

The probability of equity hitting the trigger $S^*$ can be modeled as a first exit barrier problem. Given a risk-neutral process for the equity

$$dS = (r - q)Sdt + \sigma_s SdW$$

where $r$ is the risk-free rate, $q$ is the dividend rate, $\sigma_s$ is the equity volatility, and $W$ is Brownian motion, we can show that the probability of survival is given by

$$p_{\text{surv}}(t, S^*) = \Phi \left( \frac{\mu t - \log \left( \frac{S^*}{S} \right)}{\sigma_s \sqrt{t}} \right) - \left( \frac{S^*}{S} \right)^{\frac{2\mu}{\sigma_s^2}} \Phi \left( \frac{\mu t + \log \left( \frac{S^*}{S} \right)}{\sigma_s \sqrt{t}} \right)$$

where $\mu = r - q - \sigma_s^2/2$ and $\Phi$ is the cumulative normal distribution. The trigger intensity and spread are now given by

$$\lambda_{CoCo} = -\frac{1}{t} \log (p_{\text{surv}}(t, S^*))$$

$$s_{CoCo} = \lambda_{CoCo} (1 - R_{CoCo})$$

Thus, given a CoCo bond spread, we can use (12)–(14) to back out an implied equity barrier $S^*$ that corresponds to the trigger event. We can either fix the recovery rate or allow it to be a function of the barrier:

$$R_{CoCo} = \begin{cases} 
R_{\text{fixed}} & \text{for fixed recovery assumption} \\
S^*/X & \text{otherwise} 
\end{cases}$$

An example of different recovery rates is provided in Figure 1 at the end of this section.
Random barrier problem

We can generalize the previous model \(12\)–\(13\) that was introduced by \[3\] to include uncertainty in the estimation of \(S^*\). Assume that the barrier \(S^*\) is lognormally distributed with standard deviation \(\lambda_{S^*}\):

\[
S^* = \overline{S}^* e^{z \lambda_{S^*} - \frac{\lambda_{S^*}^2}{2}}
\]  

where \(z\) is a normal random variable with mean zero and variance one. This is not an unreasonable extension given that accounting information inherently lags. We can interpret \(\lambda_{S^*}\) as the uncertainty in balance sheet information and use it as a tuning parameter for the estimation of spread volatility. In this version \(12\) becomes

\[
 p_{\text{surv}}(t, S^*) = \Phi \left( \frac{\mu t - \log \left( \frac{S^*}{\overline{S}^*} \right) + \frac{\lambda^2}{2}}{\sqrt{\sigma_s^2 t + \lambda^2}} \right) - \left( \frac{S^*}{\overline{S}^*} \right)^{2\mu \sigma_s^2} e^{C} \Phi \left( \frac{\mu t + \log \left( \frac{S^*}{\overline{S}^*} \right) - \frac{\lambda^2}{2} + 2\lambda^2 \frac{\mu}{\sigma_s^2}}{\sqrt{\sigma_s^2 t + \lambda^2}} \right)
\]

where

\[
C = \lambda^2 \frac{\mu}{\sigma_s^2} \left( -1 + 2\frac{\mu}{\sigma_s^2} \right)
\]  

Note that \(\lambda = 0\) reduces to the previous case. We provide an example in Figure 1, where the CoCo spread is plotted as a function of the equity barrier \(S^*\). As \(S^*\) increases, the spread monotonically increases.

EQUITY-TO-CREDIT (E2C)/CREDITGRADES STRUCTURAL MODEL

Under a structural model, the asset value of a firm is typically modeled as geometric Brownian motion and default occurs when the asset value dips below the liability level of the firm. The key inputs to a structural model are the asset value and asset volatility, both of which are unobservable. In a traditional implementation, one estimates the asset value through consolidated balance sheet information. We differ in our implementation in two important ways:

- Outputs of the model are risk-neutral probabilities and spreads
- Bypass the estimation of liability via balance sheet information; calibrate to the observed spread by backing out an implied leverage

The structural model we present is a first exit barrier model, similar to the equity barrier model in the previous section. See \[5\] and \[9\] for a description of the model mechanics. We present the key results of the model, which we call the Equity-to-Credit (E2C) model, below.
The survival probability of the E2C model is given by

\[ p_{\text{surv}}(t, B^*) = \Phi \left( -\frac{A_t}{2} + \frac{\log(d)}{A_t} \right) - d\Phi \left( -\frac{A_t}{2} - \frac{\log(d)}{A_t} \right) \]  

(19)

where

\[ d = \frac{V}{B^*} e^{\lambda^2} \]  

(20)

\[ A_t = \sqrt{\sigma^2 a t + \lambda^2} \]  

(21)

and the barrier follows a lognormal distribution such that \( E[B] = B^* \) and \( \lambda = \text{Stdev log } B \). The asset value and volatility are estimated as follows:

\[ V = S + B^* \]  

(22)

\[ \sigma_a = \sigma_s \left( \frac{S}{S + B^*} \right) \]  

(23)

Thus, as in the previous section, the survival probability (19) is a function of market observable inputs and the barrier. Likewise, given the spread of the CoCo bond, we can back out a barrier; in this case, an implied debt barrier.

Note that given all else equal, a CoCo with a greater spread will calibrate to an implied barrier that is greater.

\[ ^7 \text{See Finger [5] for a derivation.} \]
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